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Theory and Application of Point-Diffraction Interferometers

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The point-diffraction interferometer is an interferometer for measuring phase variations in which the reference wave is produced by a point discontinuity in the path of the beam. Its simplicity makes it very suitable for testing instruments *in situ*, and some such tests are described. The general theory shows that other diffracting apertures can be used and relates the technique to phase-contrast microscopy and to scatter-plate interferometry.

§1. Introduction

The point-diffraction interferometer,[1] PDI, belongs to the class of interferometers that measures the variations of phase across a wavefront. In such interferometers a coherent reference wave, usually spherical or plane, is made to interfere with the wave being examined. The interference shows the variations of phase difference across the waves as variations of fringe position. The PDI produces its reference wave by diffraction of some of the light at a point discontinuity placed in the path of the beam. It is a very simple instrument and has theoretical importance as a bridge between the fields of interference and diffraction.

The principle of the interferometer can be derived by the methods of spatial-filtering theory. The sum of the complex amplitudes of the wave being examined and a plane reference wave gives as its Fourier transform the amplitude image from the wave plus a δ-function. This is achieved by the filter shown in Fig. 1. The wave being examined is brought to a focus to produce an image, usually aberrated, of the point source from which it came. An absorbing film placed in the focal plane has in it the diffracting point, which can be either a small pinhole or a small opaque disk. The wave is transmitted through the film with reduced amplitude and, in addition, some light is diffracted by the point into a spherical wave. The usual adjustments of an interferometer are possible. A tilt can be produced between the wavefronts, in order to introduce straight fringes, by displacing the diffracting point laterally from the centre of the image. A longitudinal displacement out of the focal plane introduces circular fringes.

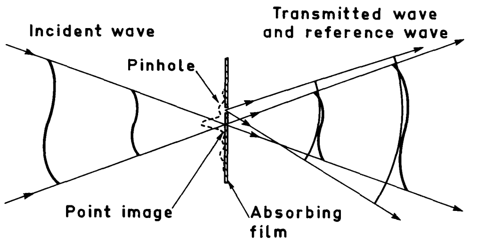

Fig. 1. Principle of the point-diffraction interferometer. An incident wave, attenuated by the absorbing film at its focal plane, interferes with a spherical wave diffracted by a discontinuity in the film.

The PDI is a common-path interferometer and has the usual advantages of this class. The fringes are very stable against vibration and a white-light source can be used. Although not required for its coherence, a laser is a very useful source since it overcomes the rather large loss of light.

The PDI is closely related to the phase-contrast test of Zernike,[2,3] in which a small diffracting disk introduces a phase change of $\pi/2$ with respect to its surround. The test is used with no tilt and the $\pi/2$ phase change increases the sensitivity to small phase variations by moving the position of zero phase away from the interference maximum.

1.1 *Pinhole size*

In practice, the diffracting region is not a point but has finite size. The amplitude of the reference wave depends on how much of the light in the image falls on this region and this varies with the aberrations of the wave and with the tilt. To produce an interference pattern of high visibility the filter should attenuate the amplitude of the direct wave to match. This indicates a disadvantage of the PDI; ideally a filter of different transmittance should be used for each tilt. Usually the optimum transmittance is low and a clear diffracting region has a larger transmittance difference than an opaque one and so diffracts more light.

The larger the diffracting region, the more the variations of visibility with tilt are smoothed out. But, if the region is too large, some of the variations of the original wave are reproduced in the reference wave. The region should not be much larger than the centre of the Airy disk that the original wave would produce if it had no aberrations.

§2. Use of the Interferometer

2.1 *Construction*

The PDI consists of an absorbing metal coating on a clear substrate. In this coating there is a clear pinhole the size of which is chosen to suit the requirements discussed in the previous section. The transmittance chosen for the coating will depend on the tilt to be used and on the phase variations to be measured, since these determine the spread of the image. In practice, transmittances between 0.005 and 0.05 have proved suitable, with 0.01 being best for most tests. If possible, a range of filters should be available, differing in pinhole size and in transmittance.

Polished glass has been used successfully for the substrate. Cleaved sheets of mica have also been used; these are freer from small surface defects and give less unwanted scattered light. Both gold and aluminium have been used for the coatings. Pinholes have been obtained by shadowing with small spheres during the evaporation of the coating or by vaporizing holes in the coating by a laser focused on it through a microscope; their size depends on the numerical aperture of the objective used.

2.2 *Laboratory applications*

The usefulness of the interferometer has become apparent in a wide variety of tests. It has been found that the variation of fringe visibility with tilt is not unduly restrictive, since in most cases this variation is small within the usual range of tilts desired for optimum display. Tests have been carried out of individual components, in single and double passage, on- and off-axis. Compound systems are no more difficult to test than single elements and, providing a point image is accessible at an

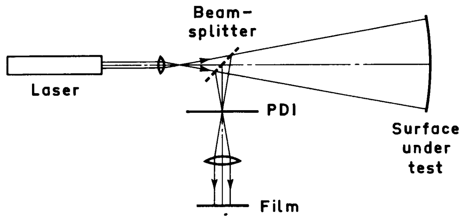

Fig. 2. Method of testing a reflecting surface with the PDI.

intermediate image plane of a multi-element system, the error up to that plane can be determined. Such an application of the interferometer has been the testing of a reflection coronagraph at three of its four successive image planes. There is no fundamental limitation on the aperture of a system which can be tested; the interferometer has been found equally useful for testing microscope objectives and large astronomical telescopes.

A configuration for testing a concave surface, using a laser source, is represented in Fig. 2. To obtain interference, the diffracting aperture must be critically adjusted to be close to the image. For this, a mount having fine orthogonal adjustments is required. The interference pattern can be projected directly on a white or ground-glass surface, even with a low-power laser. The wavefront error shown includes not only that due to the surface under test, but also that of the incident wavefront. Providing the divergent beam from the laser overfills the test aperture, the incident wavefront error is likely to be negligible.

The interferometer has also been used with a white-light source. A concentrated-arc lamp, imaged to a sufficiently small angular size, is convenient. Interferograms have good visibility when a broad-band filter is used.

2.3 *Telescope testing*

The application of the PDI to testing astronomical telescopes under operating conditions with a star source has been investigated.[4)]

The test was carried out on the Mount Palomar 152-cm telescope of the Hale Observatories. The interferometer was attached at the Cassegrain focus. Under conditions of moderately good atmospheric seeing, with the telescope set on a zero-magnitude star, it was simple to find an interference pattern with fringes well defined across the entire field;

a representative interferogram, recorded photographically without image intensification, is shown in Fig. 3. The patterns were partly blurred due to the phase variations caused by atmospheric seeing and the image movement during the exposure time of 1 s. Operationally, the test proved to be a simple, straightforward procedure.

For such a test, an accurate error analysis requires excellent seeing conditions, if only for a period of a few seconds, or a system to sample the interference pattern over an extended period of time, compute the wavefront of each sample, and then average out the randomly varying component due to the seeing.

A 61-cm Cassegrain telescope has been tested with the interferometer and a laser. It was possible to incline this telescope to an unobstructed horizontal view and a 2-mW laser was set up at a distance of 2.4 km. Its advantages over a stellar source are: (i) the much higher irradiance in the recording plane (the interference pattern could be projected directly on the inside of the telescope dome), (ii) a stationary system, and (iii) improved atmospheric seeing. Figure 4 shows such an interferogram. As well as some minor perturbations due to seeing, the interferogram reveals a substantial localized error due to mirror misalignment.

Figure 5 is an interferogram of a 20-cm Schmidt-Cassegrain telescope recorded indoors with a laser source at a distance of about 30 m. This figure illustrates the quality of telescope interferograms attainable with the PDI.

2.4 *Adjustable filter*

A PDI suitable for a wide range of tilts requires some means of adjusting the relative amplitudes of the two waves. A filter of this type has been made from polarizing film. A pinhole was made with a sharpened needle in a sheet of Polaroid unmounted K-sheet, which is about 50 μm thick. The holes had diameters between 20 μm and 50 μm. The sheet was then

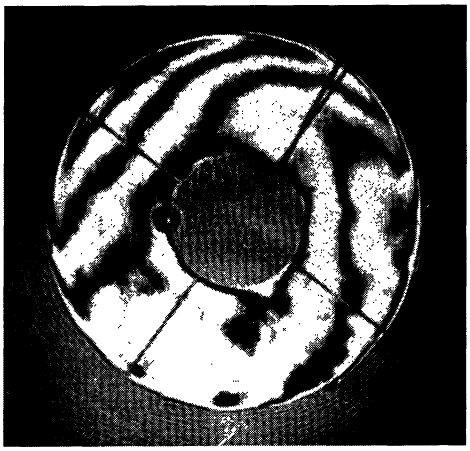

Fig. 4. Interferograms of a 0.6-m Cassegrain telescope with a laser source; exposure 1 ms.

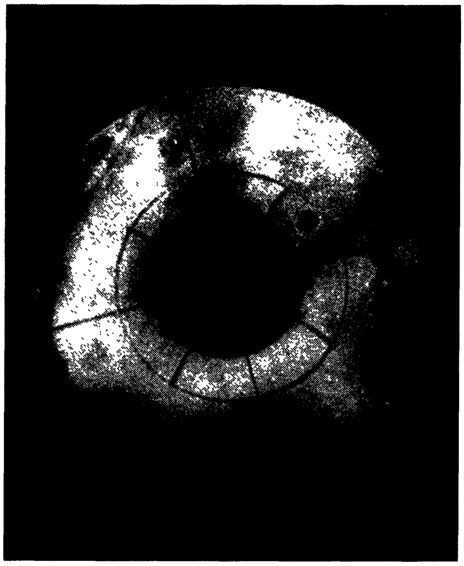

Fig. 3. Interferogram of the 1.5-m Mount Palomar telescope with a star as source.

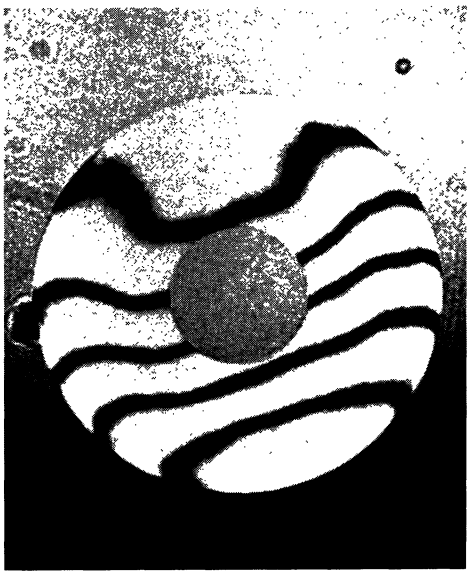

Fig. 5. Laboratory test of a 20-cm Schmidt-Cassegrain telescope.

mounted between glass plates with a liquid that matched the refractive index of the Polaroid, about 1.50. This filter was used with a polarized laser as source and followed by a polarizer that could be rotated to vary the amplitude ratio. High-contrast interferograms have been obtained over a range of tilts.

§3. Extended Apertures

It has been assumed that the wave being examined comes from a point source and that the diffracting aperture, if not a point, is at least an approximation to one. But the PDI can be used with an extended incoherent source and an aperture of the same shape, provided the spread function of the aperture is broad. Each point of the source produces its own wave and from this a reference wave is diffracted by the aperture. Each such pair of waves produces an interference pattern. To obtain a satisfactory interferometer, the patterns from all points at the source should coincide; this condition is satisfied in the far field of the diffracting aperture.

3.1 *General theory*

A general theory for a PDI with an extended source can be worked out for the optical system commonly used for spatial filtering, shown in Fig. 6. In succession there are planes of the source, object, pupil, and image and points on these are represented by vectors $\boldsymbol{u}$, $\boldsymbol{x}$, $\boldsymbol{u}'$, and $\boldsymbol{x}'$. An optical system images the source to the pupil; another forms the final image of the object.

The units of length of the vectors $\boldsymbol{x}$ and $\boldsymbol{x}'$ are the radii of the object and image fields, both of which are assumed to be circular. If the edge of the object subtends angles θ and θ' at the centres of the source and pupil, the magnitudes of the vectors $\boldsymbol{u}$ and $\boldsymbol{u}'$ are related to actual distances in these planes by the factors $(2\pi/\lambda)\sin\theta$ and $(2\pi/\lambda)\sin\theta'$; they are the usual diffraction units. The two-dimensional Fourier transform that represents Fraunhofer diffraction is then

$$F(\boldsymbol{x})=\frac{1}{2\pi}\iint_{-\infty}^{+\infty} f(\boldsymbol{u})\exp(-\mathrm{i}\boldsymbol{u}\cdot\boldsymbol{x})\mathrm{d}\boldsymbol{u}. \qquad (1)$$

A point source at $\boldsymbol{u}$ produces an amplitude $\exp(-\mathrm{i}\boldsymbol{u}\cdot\boldsymbol{x})$ at the object. This is taken to have an amplitude transmittance

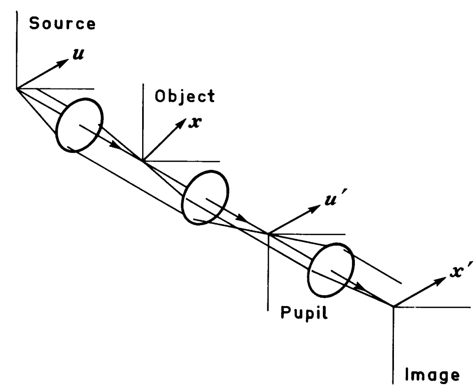

Fig. 6. Notation for the general theory.

$$L(\boldsymbol{x})=\exp[\mathrm{i}\psi(\boldsymbol{x})]; \qquad (2)$$

it has phase variations only. The amplitude at the entrance pupil is then $l(\boldsymbol{u}'-\boldsymbol{u})$.

The filter that constitutes the PDI is at the pupil. It has an amplitude transmittance of the form

$$t_1 p(\boldsymbol{u}')+t_2 g(\boldsymbol{u}'-\boldsymbol{U}'). \qquad (3)$$

Here t_1 and t_2 are constants and $p(\boldsymbol{u}')$ and $g(\boldsymbol{u}')$ functions that represent the full aperture of the pupil and the diffracting aperture, when the latter is centred. Both functions are unity over their apertures and zero outside them. The tilt is given by $\boldsymbol{U}'$.

The complex amplitude at the image plane then has the form

$$A(\boldsymbol{x}')=t_1 A_{\mathrm{p}}(\boldsymbol{x}')+t_2 A_{\mathrm{g}}(\boldsymbol{x}')\exp(-\mathrm{i}\boldsymbol{U}'\cdot\boldsymbol{x}'), \qquad (4)$$

where A_{p} and A_{g} are the amplitude images formed by each aperture on its own. When the pupil is large we can write

$$A_{\mathrm{p}}(\boldsymbol{x}')\approx L(\boldsymbol{x}')\exp(-\mathrm{i}\boldsymbol{u}\cdot\boldsymbol{x}'); \qquad (5)$$

the image has uniform amplitude and the same phase variations as the object. $A_{\mathrm{g}}(\boldsymbol{x}')$ will also have an amplitude that is approximately uniform, but a different phase.

The interference from this source point is the squared modulus of eq. (4). That for the whole source, which is assumed to be incoherent and of shape $g(\boldsymbol{u})$, the same as the diffracting aperture, is the integral of this interference over $g(\boldsymbol{u})$. It is

$$B(\boldsymbol{x}')=|t_1|^2+|t_2|^2+2\mathrm{Re}\,\{t_1 t_2^* L(\boldsymbol{x}')M^*(\boldsymbol{x}') \times\exp(-\mathrm{i}\boldsymbol{U}'\cdot\boldsymbol{x}')\}, \qquad (6)$$

where Re denotes the real part, * the complex conjugate, and

$$M(x') = \frac{1}{2\pi} \iint_{-\infty}^{+\infty} |G(x'-x)|^2 L(x) \times \exp(-iU' \cdot x) dx, \quad (7)$$

$G(x)$ being the transform of $g(u)$. The transform of $M(x')$ is

$$m(u) = h(u) l(u - U'), \quad (8)$$

where $h(u)$ is the autocorrelation of $g(u)$ and $l(u)$ the transform of $L(x)$.

Equation (6) has the usual form for two-beam interference. The visibility depends on the modulus of M and the phase includes arg M. Hence, for a source $g(u)$ to give a suitable interference pattern, the corresponding function M should be substantially uniform in amplitude and phase over the field of view.

For a point source, $g(u) = 2\pi\delta(u)$; its autocorrelation function $h(u)$ is also a δ-function, and so is $m(u)$. $M(x')$ is thus the constant $l(U')$; it has uniform amplitude and phase. It appears, therefore, that satisfactory sources should have an autocorrelation function that approximates to a δ-function.

Many sources can be found with an autocorrelation function that is a δ-function superimposed on a widely spread background. On transformation this background gives a peak at the centre, which, through the convolution, eq. (7), will give some of the phase variations of $L(x')$ to the reference wave, which $M(x')$ represents. Hence any autocorrelation background must be small.

3.2 *Random pinhole array*

A function $g(u)$ that consists of a set of randomly spaced pinholes should be suitable, provided they are widely spaced. A PDI with such a source and aperture appears to resemble a scatter-plate interferometer,[5] which uses two identical plates with randomly varying phase changes. In the latter, however, the interference is between the beam transmitted by one plate and diffracted by the other and that diffracted, then transmitted. The transmitted-transmitted beam represents stray light that must be removed by a stop; the diffracted-diffracted beam is of negligible intensity.

Since the first random-pinhole plate of a PDI has an opaque background, it gives no directly transmitted beam from it. The interference is now between the doubly diffracted beam, previously neglected, and the diffracted-direct beam.

A matched source and screen, produced by photographic reduction of a computer plot, gave interference fringes in a low-aperture optical system but the quality was poor, mainly because of the difficulty of registering the source image on the aperture.

3.3 *Line sources*

Patterns made up of narrow lines have also a suitable autocorrelation function. Two of these have been used: a narrow annulus and a narrow cross. Both had lines of width 1.7 in units of $|u|$.

The cross gave similar difficulties to those found with the random pinholes but the annulus was simple to adjust and use. This suggests that the most difficult part of registering the two patterns involves their relative orientations. For the annulus a match in size is readily ensured by the use of matched lenses in the optical system. The result obtained is shown in Fig. 7.

The annular source gives much brighter interferograms than the point. It could be used to make a simple white-light interference microscope that would show less diffraction effects than a point source.

The phase-contrast microscope now appears as a PDI with a wide annulus and no tilt. The traditional explanation is in terms of the interference between a direct beam through the object that passes completely through the phase ring and the beam diffracted by the object which passes around the ring. Our theory gives the alternative explanation as the interference of the whole wave from the object as diffracted

Fig. 7. Fringes given by a PDI with a narrow annulus as source and diffracting aperture.

by the full aperture with the same wave as diffracted by the phase ring.

Since the phase ring is not small compared with the full aperture, the approximation of eq. (5) cannot be used. The amplitude images, given in eq. (4), are convolutions of the object wave and the spread functions of the two apertures. Neither aperture is narrow, so both have relatively narrow spread functions that differ only in their finer detail. The convolutions then differ only where the object shows fine-detail variations; over regions of slowly varying phase there is little difference. Thus, as is well known but not well explained by the traditional explanation, phase contrast shows clearly edges and similar rapid variations of phase, but not the phase differences between fairly large regions.*

*An interferometer of this style has been described by S. Mori, Trans Soc. Instrum. and Control Engng **8** (1972) 125 [in Japanese], using a point as source and a wire as the diffracting region.

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References

1) R. N. Smartt and J. Strong: J. Opt. Soc. Amer. **62** (1972) 737.
2) F. Zernike: Physica **1** (1934) 689.
3) C. R. Burch: Monthly Not. Roy. Astron. Soc. **94** (1934) 384.
4) R. N. Smartt: J. Opt. Soc. Amer. **64** (1974) 558.
5) J. M. Burch: Nature **171** (1953) 889.